DOCUMENT RESUME

ED 090 038 SE 017 628

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TITLE The Use of Manipulative Lessons in Primary Grade Arithmetic in a Program for Individualized Instruction.
PUB DATE Apr 74
NOTE 16p.; Paper presented at the annual meeting of the American Educational Research Association (Chicago, Illinois, April 1974)

EDRS PRICE MF-$0.75 HC-$1.50 PLUS POSTAGE
DESCRIPTORS *Curriculum Development; *Elementary School Mathematics; Evaluation; *Individualized Instruction; Individualized Programs; *Manipulative Materials; Program Descriptions; Program Effectiveness; *Program Evaluation
IDENTIFIERS Research Reports

ABSTRACT

A description and evaluation are given of the Individualized Mathematics (IM) program. IM is an outgrowth of the work of the Learning Research and Development Center (LRDC) at the University of Pittsburgh. The program has developed and implemented manipulative lessons for primary grade (K-3) students. Like other LRDC programs, this one includes such components as specific instructional objectives, lesson materials to teach each objective, a testing program for monitoring pupil progress, and specific procedures for using these components in the individualized instruction. The emphasis in this program is on the use of manipulative materials for lessons in which the students have a high degree of self-management. The results of two years of development and investigation showed that the program (1) was manageable, (2) permitted independent study, (3) facilitated pupil progress, (4) produced desired achievement test results, and (5) needed modification to make costs and storage requirements more feasible. (JP)

ED 090038

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The Use of Manipulative Lessons in Primary Grade Arithmetic in a Program for Individualized Instruction [1]

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1974

The work reported herein was supported by the Learning Research and Development Center, supported in part by funds from the National Institute of Education (NIE), Department of Health, Education, and Welfare. The opinions expressed do not necessarily reflect the position or policy of NIE and no offical endorsement should be inferred.

[1] A paper prepared for presentation at the 1974 AERA meeting held in Chicago April 15-19.

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The Use of Manipulative Lessons in Primary Grade Arithmetic in a Program for Individualized Instruction

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The development effort that is the focus of this report is a part of a larger project of the Learning Research and Development Center, namely the development, over the past three years, of the Individualized Mathematics (IM) program for use in kindergarten through third grade. However, the writers feel that the work on the development and implementation of a systematic procedure for the use of manipulative materials in a rather extensive program of individualized instruction should be of enough general interest to warrant its being reported as a major project in itself.

The Individualized Mathematics (IM) program is a system of instruction consisting of such components as specific instructional objectives, lesson materials to teach each objective, a testing program for monitoring pupil progress, and specific procedures for using these components in the individualization of instruction. IM can be considered as growing out of two earlier LRDC programs, namely IPI Math (Lindvall and Bolvin, 1967) and PEP Quantification (Wang, Resnick and Schuetz, 1970). IPI Math is a system of individualized instruction that relies heavily on pupil use of special workbook materials, on paper-and-pencil tests, and on individualized pupil prescriptions developed on the basis of diagnostic testing. The PEP Quantification program used in kindergarten and grade 1, introduced procedures for using manipulative lessons (the "PEP boxes") in individualized instruction. PEP has depended heavily upon

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[1] Major contributors to the design and development of the manipulative lessons used in this study included the following members of the LRDC Math Project staff: Alice Hosticka, M. Kathryn Meese, George R. Miller, and Karen Lake.

having the teacher produce the boxes needed in her classroom. This latter procedure has a positive effect in that it gives the teacher an intimate knowledge of the lesson materials, a type of knowledge that is of great value both in teaching and prescribing. However, the procedure also has certain drawbacks, such as the fact that the quality and quantity of lesson materials varies greatly from room to room and that general dissemination of the program requires rather intensive training of teachers in how to design lessons. It was felt that a program designed for widespread use should place less reliance on teacher production of lesson materials and also insure the availability of at least the minimum set of exercises required to teach each objective. Teacher creativity could then be addressed to the task of supplementing the basic program.

One way of describing the task of the IM project staff, then, is to say that we sought to take the type of manipulative lessons introduced by PEP, develop them in a somewhat standard format, design enough of these lessons to teach the IM objectives, standardize the prescription development procedures, and take whatever other steps were necessary to enable teachers to use such lesson materials in the same systematic fashion employed with published IPI booklets. Our question was "Can we incorporate manipulative lessons into individualized instruction under a planned system which permits its easy adoption by typical teachers?"

Basically, then, this paper reports results of a three-year development effort designed to investigate procedures for permitting primary grade pupils, working in an individually prescribed instruction program to use manipulative lessons under conditions involving a high degree of self-management.

Objectives

This program, while ultimately concerned with the improvement of pupil mastery of arithmetic content, had as its initial goals the development

of an instructional system that gave pupils the desired manipulative experiences and that was manageable by pupils and teachers working in an on-going school program. Achievement of these goals, that is, demonstrating the feasibility of this type of system and materials, would then set the stage for subsequent intensive work on the design of improved lessons. Specific objectives for this first phase of the project were to develop a system having the following qualities:

1. Materials and procedures will be such that teachers, working in an individually prescribed instruction classroom, will be able to manage their implementation.

2. Pupils will be able to utilize materials and procedures with only the minimum type of teacher direction typically required in the operation of individualized programs.

3. The manipulative materials, when used under this individualized procedure, will provide pupils with learning experiences that enable them to master instructional objectives as evidenced by the ability to pass unit posttests.

4. Pupils using these manipulatives and procedures will perform as well or better, on end-of-year standardized achievement tests, than pupils studying in the same program before the introduction of such materials.

5. The use of these procedures will be found to be feasible from the standpoint of cost, storage, and upkeep.

Theoretical Framework

The type of manipulative exercises designed for the program were based largely on materials and activities suggested by the work of Dienes (1960), Piaget (1965), and Bruner (1966). Procedures for their incorporation into an individualized instructional system were derived from the

work of Resnick and Wang (Wang, Haberman and Maganau, 1973). Successful implementation of these lessons in the classroom demands teacher attention to the shaping of pupil "learning-to-learn" behavior through continuing positive reinforcement of appropriate study activities and successful performance of manipulative tasks (Leinhardt, 1974).

Procedures

Like its predecessor programs, IPI and PEP, the IM program is based on specific instructional objectives organized into units. It involves a total of 45 units (identified by numbers 1-45) and 243 objectives (with each objective identified by a unit number and letter). Each manipulative exercise developed for the program is designed to teach one objective or a part of one objective and is packaged in a small box (dimensions 13" x 7" x 2"). With most objectives, the IM program also provides a lesson booklet containing paper-and-pencil exercises as an additional type of study material. Each box of manipulatives is designed to give the pupil concrete experiences with the type of activity specified by the objective; experiences in counting, pairing, exchanging, combining sets, removing sub-sets, and so on. Each box is identified by unit number, by objective letter, and box number for that objective. For example, the box labeled M13B1 is designed to teach objective B in unit 13 and is the first box for that objective.

In order to maximize the chances that primary grade pupils could make effective use of these boxes of manipulative exercises, on an independent study basis, major attention has been given to four components of the system, (1) prescription tickets, (2) steps for pupil use of boxes, (3) box-lid models, and (4) instructions for the teacher.

1. Prescription Tickets. A pupil notes what box he is to work with by referring to his prescription "ticket." A sample of what appears on such a ticket is shown in Figure 1. The teacher indicates exactly what boxes the pupil is to use by placing a mark in the first square following the box designation. When the student sees this mark on his prescription ticket, he goes to the set of shelves where the boxes are stored and selects the box prescribed. He then returns to his seat or other workspace and begins his study. When he has completed his work with the box, he will be responsible for returning it to the shelf and filing it in the proper order.

Of course, the prescription ticket is an essential guide for both the teacher and the pupil. Since it lists all the materials available for the study of each objective, it is an important reminder to the teacher of those things she might prescribe. In our ticket for Unit 13 there are four boxes (M13A1, M13A2, M13A3, M13A4) and one booklet (M13A) available for teaching objective A. The teacher might prescribe all, or only some, of these materials, depending upon her knowledge of a pupil's needs. The squares following each box or booklet identification are used to indicate what is prescribed and also to record successful pupil completion of the assignment. Both teacher and student quickly become proficient in marking and interpreting the ticket. Since this ticket is the means by which the pupil determines exactly what he is to study and the record form upon which his daily progress is indicated, it is a key component of the system that facilitates the effective use of boxes on an individualized basis.

2. Steps for Use of Boxes. When a pupil starts to study, using the box with its manipulative exercise, he should follow four steps that he has been taught as representing the correct procedure for this type

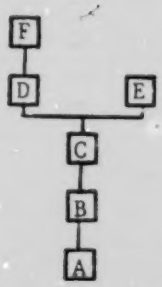
INDIVIDUALIZED MATHEMATICS

UNIT 13 Groups by Tens and Ones

Mastery

Date	Days	Teacher

Name _____ Class _____ Student No. _____



A

M-13-A-1

M-13-A-2

M-13-A-3

(G) M-13-A-4

M-13-A

Unit Test Scores

Skill	Points	Pre	%	Post	%	Post	%	Post	%
A	2								
B	3								
C	4								
D	4								
E	3								
F	5								
Test Dates									

CET

Date	Sc.	%	Date	Sc.	%

B

M-13-B-1

M-13-B-2

M-13-B-3

M-13-B-4

(G) M-13-B-5

(G) M-13-B-6

M-13-B

CET

Date	Sc.	%	Date	Sc.	%

Figure 1. Portion of Prescription Ticket for Unit 13 of IM Program

of study (Leinhardt, 1974). These steps are:

(1) "Remove all materials from the box and place them in front of you." (In doing this the student will probably examine any unfamiliar objects and become somewhat acquainted with what is in the box. He may also "organize" them in some way that makes sense to him.)

(2) "Study the picture or model shown on the inside of the box lid and use materials from the box to duplicate this model. When this is completed, ask the teacher to check your work." (When the pupil has done this correctly, the teacher will praise him for being correct, explain the operation or relationship that it demonstrates, and tell him to carry out the same operation with the other comparable sets of materials in the box.)

(3) "Carry out this same operation with all other sets of materials." (This represents practice, on the part of the student, in carrying out the operation learned in step 2. Of course, additional teacher or peer tutoring may be required here.)

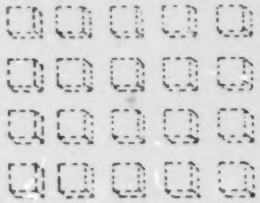
(4) "Have the teacher check your completed work." (Here the teacher may find it necessary to do some additional tutoring and require the pupil to repeat some portion of the exercise. She should also ask the questions listed inside the box lid.)

The IM experience indicates that if the student employs these steps, he can use these boxes on an independent basis and can acquire mastery of objectives through such learning activities.

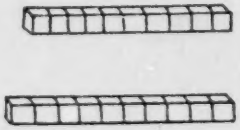
3. Box-lid Models. Figure 2 shows an example of the picture and other explanatory material found inside the box lid. The material found in the upper left-hand portion of the figure is essentially a facsimile of

Box: M 13 A 2

Get this many Dienes Blocks.



Exchange each set of 10 for a tens bar.



Inventory:

4 cards
65 Dienes Ones Blocks
8 Dienes Tens Bars

Questions:

1. How many blocks are in each group?
2. How many groups of Dienes blocks do you have? Count the groups.
3. Can you exchange the groups of ones for a tens bar? Count the tens.

Teacher Instruction for Student:

The student places Dienes blocks on the pictures, then groups the blocks into groups of tens, and exchanges each group for a tens bar.

Figure 2 Inside Cover Model for IM Box M13A2

a card that the pupil should be able to find in the box. Since this particular lesson is typically used in the latter half of first grade, a pupil may not be able to read what is on the card. If he cannot read, the pictures may provide sufficient cues. If not, he may need to have the first card explained to him by the teacher or another student.

4. Instructions for Teacher. Note, too, that the information on the inside of the lid provides other assistance to the teacher, (1) a verbal explanation of what the pupil is to do, (2) an inventory list of what should be found in the box, and (3) some questions to be used in interacting with the student. This type of assistance for the teacher is particularly helpful in view of the fact that during the course of one class period she may have to offer guidance and instruction with respect to thirty or more different boxes. It is also an aid in assuring that all pupils have certain minimum experiences, in common, when they use a given box.

Implementation

This project was carried out within the context of a large, inner city elementary school in Pittsburgh and in a medium size elementary school in a Pittsburgh suburb. The materials were used in 20 classrooms in grades K, 1, 2, and 3. Work in this large number of classrooms, dictated partly by over-all development commitments of LRDC, while causing problems with certain aspects of formative evaluation, permitted an investigation of problems of logistics and school-wide management that would not have been possible in a study confined to one or two classrooms. In total, 390 manipulative exercises were produced and used in these classrooms. Of course, more manipulatives were produced for the lower levels of the curriculum than for the higher levels, where a greater dependence was placed on lesson booklets. Manipulative lessons

were developed for those units and objectives where it was judged that learning could be enhanced by pupil manipulation of appropriate materials. The relatively large number of manipulatives (boxes) involved was again a result of the commitments cited above. This too, facilitated the study of many practical problems of classroom and school-wide management. Work in two different schools permitted the trial of two different procedures for storage and pupil access to boxes.

Results and Conclusions

In considering the results of this study or project it is important to keep in mind the fact that it is in no sense an experimental study or an evaluation of the effects of the specific lessons used. The goal of this study was to design and implement a set of procedures that permitted the extensive use of manipulative lessons in a program of individualized instruction. The specific lesson materials used here, although found to be somewhat effective in this study, are now the focus of continuing development work for the project staff. Prior to pursuing this lesson refinement activity, however, it was deemed essential to determine the operational feasibility, in individual classrooms and in overall school operations, of the procedures involved. Such was the purpose of this study.

Results can be summarized in terms of each of the five major objectives of the project.

1. The IM manipulative lessons have been used for three years in 20 classrooms in two elementary schools. The average size of these classes was approximately 25 students and each teacher had the help of one aide or an assistant teacher. As could be expected, the first year of implementation involved extensive formative evaluation of the materials, carried out through classroom observation, individual try-

outs with pupils, and a variety of provisions for teacher feedback. After the modifications resulting from this evaluation process had been made, the overall system was found to be manageable by all teachers involved. Teachers who, in the preceding years, had attempted to produce their own "boxes" were particularly impressed with the relative simplicity of the new system. The major "datum" concerning the manageability of the system is the fact that the program is now operating in these classrooms with no involvement of IM Project personnel.

2. Most of the manipulatives have been found to be manageable, by the typical student, on an independent study basis. Information concerning the achievement of this goal was obtained through reports from teachers and through classroom observation by IM staff members. During the course of this project several boxes were revised to make them more useable in this way. A rather common type of revision was the simple one of removing some of the practice exercises so as to make it easier for the pupil to carry out his initial organization of the contents. Of course, in a few instances teachers used the box materials as the basis for small group instruction.

3. At this stage of development pupils using these manipulatives are mastering unit posttests at approximately the same rate as students using previous versions of the IMI program not involving such materials. Reduction in number of required manipulative exercises (to be implemented in 1973-74) may result in more rapid progress.

4. As indicated earlier, higher levels of pupil achievement on standardized tests was not established as a goal of the present project. However it was deemed important to demonstrate that the use of these manipulatives did not result in poorer preformance on those tests typically used in the schools involved. Results from end-of-year adminis-

trations of the Stanford Achievement Tests and the Wide Range Achievement Tests are summarized in Tables I and II.

Table I

Mean Scores for Arithmetic Sub-tests on the Stanford Achievement Tests for Pupils in Grades 1-3, 1968-1973

School A

Grade Level	Sub-Test	School Year				
		68-69	69-70	70-71	71-72	72-73
Grade 1	Arith.	2.2	2.4	2.4	2.5	2.5
Grade 2	Arith. Comp.	2.8	2.7	2.9	3.0	2.9
	Arith. Concepts	3.2	3.0	3.4	3.7	3.2
Grade 3	Arith. Comp.	3.4	3.7	4.1	3.8	3.9
	Arith. Comcepts	2.5	2.9	3.2	4.8	4.7

Table II

Mean Scores for the Arithmetic Sub-test on the Wide Range Achievement Test for Pupils in Grades 1-3, 1969-1973

School B

Grade Level	School Year			
	69-70	70-71	71-72	72-73
Grade 1	2.1	2.4	2.4	2.5
Grade 2	2.3	2.7	2.9	2.8
Grade 3	3.1	3.2	3.0	3.6

These scores, and the conditions under which they were obtained, permit only the limited generalization that during the years in which the new manipulative lessons were used (70-71, 71-72, 72-73) pupil performance was at least as good as in preceding years. Due to the fact that other innovative programs were also being used in these schools during the years in question, any higher scores for the last three years cannot be attributed to the use of manipulatives.

5. The only negative evidence concerning the feasibility of the use of manipulative lessons of this type arose in the areas of materials storage and maintenance. With respect to materials storage and procedures by which pupil secure lessons, a different system was used in each of the two development sites. In School A it was possible to store all boxes (including 5 copies of each box) in one central materials center and to have all pupils go to this center to obtain and return each box as it was used. In School B, since more classrooms were involved and since they were spread over three floors of the building, it was necessary to keep a supply of all potentially needed boxes (1 copy of each box) in each classroom. The latter procedure poses problems both in terms of the number of boxes that have to be produced and in terms of classroom storage space. In an effort to deal with these problems, preparations for 1973-74 have included converting many of the box materials into laminated booklets. This was done with those boxes that use some type of card as a guide for what is to be done and that also use manipulative materials that are the same as those used in other lessons. This latter circumstance permits the storing of these common manipulatives (e.g., counting cubes, Dienes blocks, plastic number lines) in a central storage location in the classroom. The cards from each such box are then laminated and bound together with a spiral plastic binding to constitute a "laminated booklet." This has reduced both cost and storage space. Solutions to other

problems associated with storage and maintenance are now being pursued by the LRDC's School Implementation Staff. Of course, it is assumed that many of these problems, including that of cost, can be solved if and when the materials are produced by a commerical publisher. Precedent for this is found in the experience of the LRDC Science Project in going to the commercial production of their manipulative exercises. It should also be pointed out that the IM manipulatives are non-consumable and initial costs cover the materials needed for several years of operation.

Summary

Previous research and development work on the use of manipulatives with early study in arithmetic has typically involved group instruction with continuous supervision by a teacher or relatively free play where activities are not directed toward pupil mastery of specific learning goals. The current emphasis on individualizing instruction has resulted in many developers facing the problem of how to incorporate such activites into a structured individualized system. This study developed detailed procedures and a considerable quantity of lesson materials to meet this problem. These procedures were found to be quite effective and manageable and could be adapted for use in a variety of programs for individualized instruction. The study also discusses a number of practical problems that would be faced by the persons implementing the over-all system. Completion of this phase of the project now provides a context for intensive work on the design of improved manipulative lessons.

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